CTP-TAMU-29/95
ACT-11/95
hep-ph/9508253

The Missing Doublet Model Revamped

Jorge L. Lopez[1,2,†] and D.V. Nanopoulos[1,2]

[1]Center for Theoretical Physics, Department of Physics, Texas A&M University
College Station, TX 77843–4242, USA
[2]Astroparticle Physics Group, Houston Advanced Research Center (HARC)
The Mitchell Campus, The Woodlands, TX 77381, USA

Abstract

We revisit the Missing Doublet Model (MDM) as a means to address the apparent difficulties of the minimal $SU(5)$ supergravity model in dealing with the doublet-triplet splitting problem, the prediction of $\alpha_3(M_Z)$, and the proton lifetime. We revamp the original MDM by extending its observable sector to include fields and interactions that naturally suppress the dimension-five proton decay operators and that allow see-saw neutrino masses. We also endow the model with a hidden sector which (via gaugino condensation) triggers supersymmetry breaking of the desired magnitude, and (via hidden matter condensation) yields a new dynamical intermediate scale for the right-handed neutrino masses ($\sim 10^{10}\,\mathrm{GeV}$), and provides an effective Higgs mixing parameter μ. The model is consistent with gauge coupling unification for experimentally acceptable values of $\alpha_3(M_Z)$, and with proton decay limits even for large values of $\tan\beta$. The right-handed neutrinos can be produced subsequent to inflation, and their out-of-equilibrium decays induce a lepton asymmetry which is later reprocessed (via sphaleron interactions) into a baryon asymmetry at the electroweak scale. The resulting see-saw neutrino masses provide a candidate for the hot dark matter component of the Universe ($m_{\nu_\tau} \sim \mathcal{O}(10\,\mathrm{eV})$) and are consistent with the MSW solution to the solar neutrino problem. We finally compare the features of this traditional GUT model with that of the readily string-derivable $SU(5)\times U(1)$ model, and discuss the prospects of deriving the revamped MDM from string theory.

CTP-TAMU-29/95
ACT-11/95
August 1995

[†] Address after August 15, 1995: Department of Physics, Rice University, Houston, TX 77005, USA.

1 Introduction

The much heralded convergence of the Standard Model gauge couplings in supersymmetric Grand Unified Theories (GUTs) [1], is continually being challenged by ever more precise LEP measurements of the gauge couplings. It was realized early on that the effect of the GUT particles responsible for the onset of the unified theory, was not negligible [2]. However, because of the presumed great uncertainty in the GUT physics, such discussions have been largely carried out in the context of the minimal $SU(5)$ supergravity model [3]. Central to the study of these issues is the technical point of how exactly these GUT (or lighter) particles decouple at scales below their masses. Recent investigations [4] reveal that a "smooth" decoupling of particles leads to noticeable differences from the step-function approximation. Moreover, these new effects coupled with the latest LEP data on $\sin^2\theta_W$ and the determination of the top-quark mass, lead to a greatly increased prediction for $\alpha_3(M_Z)$ [5, 6, 7], strongly suggesting that minimal $SU(5)$ GUT thresholds are unable to bring the $\alpha_3(M_Z)$ prediction down to the experimentally acceptable range [5]. This impasse may be resolved with a significant contribution from Planck-scale non-renormalizable operators [8, 7], although such effects call into question the whole field-theoretical approximation to the gauge coupling unification problem.

Even if Planck-scale physics can resolve the present α_3 discrepancy in minimal $SU(5)$, this GUT model suffers from a well known fine-tuning [9] regarding the solution of the doublet-triplet splitting problem of the Higgs pentaplets. At least three solutions to this problem (all involving non-minimal GUT models) have been proposed: the missing-partner mechanism [10], the sliding-singlet mechanism [11], and the pseudo-goldstone-boson mechanism [12]. In the sliding-singlet mechanism radiative corrections destroy the gauge hierarchy [13], whereas an additional global or local $SU(6)$ symmetry is required in the pseudo-goldstone-boson mechanism. It is very suggestive that the same investigations that uncover the α_3 discrepancy in minimal $SU(5)$, also show that in the so-called Missing Doublet Model (MDM) [10], which has as its central component the missing-partner mechanism, the α_3 prediction is decreased to acceptable values [6, 5]. As we discuss below, some variants of the missing-doublet model [14, 15] also solve the problematic situation with dimension-five proton decay operators in minimal $SU(5)$, which require the Higgs triplet mass (M_{H_3}) to exceed the GUT scale and the supersymmetric spectrum to be tuned in specific ways [16, 17], especially when cosmological constraints are simultaneously enforced [18]. In fact, an updated analysis has recently shown [19] that the upper bound on the Higgs triplet mass from unification constraints (*i.e.*, $M_{H_3} \propto e^{-5\pi/3\alpha_3},\ \alpha_3 < \alpha_3^{\rm max}$), and the corresponding lower bound from proton decay constraints (*i.e.*, $\tau_p \propto M^2_{H_3},\ \tau_p > \tau_p^{\rm min}$) are very close to each other, leaving only a small window of allowed parameter space in minimal $SU(5)$. Note also that, because of the rather large representations introduced in the MDM ($\mathbf{75},\mathbf{50},\overline{\mathbf{50}}$), it is necessary to assign some of these Planck-size masses, in order to avoid the onset of a strongly-interacting GUT below the Planck scale [5, 15]. Thus, Planck-scale physics is again unavoidable in this more realistic version of $SU(5)$ GUTs.

$SO(10)$ GUTs [20, 21] have also received a great deal of attention lately [22, 23, 24, 25], with interesting successes in the area of quark and lepton masses and mixings, although the $\tan\beta = \mathcal{O}(50)$ prediction requires fine-tuning of the supersymmetric spectrum [26, 27] to reconcile it with radiative electroweak breaking. Assuming universal soft supersymmetry breaking, the further constraints from $B(b \to s\gamma)$ and cosmology strongly disfavor the model [28]. However, most of these shortcomings are overcome when one allows certain classes of non-universal scalar masses [29, 28]. More to the point, the successes of $SO(10)$ rely on the existence of certain non-renormalizable operators (as originally suggested in Ref. [30]) that are presumed to be obtained from a string-derived model at the Planck scale. Despite initial claims [31], no *consistent* $SO(10)$ GUT string model has been derived in the context of free-fermionic strings [32]. However, these failed attempts have been enough to fuel a series of "string-inspired" $SO(10)$ GUT models [25], which are limited to certain type and number of representations (those allowed by the level-two Kac-Moody construction[1]), forcing model builders to rely heavily on postulated effective non-renormalizable operators [25]. Level-two string $SO(10)$ GUT models have been consistently constructed in the context of symmetric orbifolds [34], but with limited phenomenological success, especially in dealing with the doublet-triplet splitting problem.

In view of its field-theoretical successes, in this paper we revisit the missing-doublet model as a well-motivated, realistic contender for a grand unified model. We first review the original MDM and its features and shortcomings (Sec. 2). We then propose a simple extension of the model to naturally suppress dimension-five proton decay operators (Sec. 2). Our most substantive contribution is to endow this *supergravity* model with a hidden sector containing gauge and matter degrees of freedom (Sec. 3). Hidden sector gaugino condensation triggers supersymmetry breaking which, as we discuss, can be of the desired magnitude for suitable choices of the hidden gauge group and hidden matter spectrum. The matter condensates provide a new dynamical intermediate scale which, via non-renormalizable interactions, generates a low-energy Higgs mixing term μ. With the introduction of right-handed neutrinos to the model, this scale also becomes their mass scale, which provides a suitable see-saw spectrum of neutrino masses (Secs. 2,5). We show that the model is consistent with gauge coupling unification for experimentally acceptable values of $\alpha_3(M_Z)$ and that dimension-five proton decay operators are consistent with present limits even for large values of $\tan\beta$ (Sec. 4). Also, the out-of-equilibrium decays of the right-handed neutrinos subsequent to inflation produce a lepton asymmetry which is re-processed into a baryon asymmetry by strongly-interacting Standard Model effects (sphalerons) at the electroweak scale (Sec. 5). We finally compare the features of this traditional GUT model with that of the readily string-derivable $SU(5)\times U(1)$ model, and discuss the prospects of deriving the revamped MDM from string theory (Sec. 6). We summarize our conclusions in Sec. 7.

[1]At level two, the allowed unitary massless representations are $\mathbf{1}$,$\mathbf{10}$,$\mathbf{16}$,$\overline{\mathbf{16}}$,$\mathbf{45}$,$\mathbf{54}$ [33]. A string model containing the $\mathbf{126}$,$\overline{\mathbf{126}}$ representations used in traditional $SO(10)$ model building requires an unlikely level-five construction [33].

2 The observable sector

The original MDM [10] can be described by the following set of observable sector fields: Σ (**75**), θ (**50**), $\bar{\theta}$ ($\overline{\mathbf{50}}$), h (**5**), $\bar{h}$ ($\overline{\mathbf{5}}$), $F_{1,2,3}$ (**10**'s), $\bar{f}_{1,2,3}$ ($\overline{\mathbf{5}}$'s), interacting via the superpotential

$$W = \tfrac{1}{2}M_{75}\,\mathrm{Tr}\,\Sigma^2 + \tfrac{1}{3}\lambda_{75}\,\mathrm{Tr}\,\Sigma^3 + \lambda_4\,\bar{\theta}\Sigma h + \lambda_5\,\theta\Sigma\bar{h} + M_{50}\,\theta\bar{\theta} + \lambda_2^{ij}F_iF_jh + \lambda_1^{ij}F_i\bar{f}_j\bar{h}\ . \quad (1)$$

The expectation value of the **75** can be chosen such that the $SU(5)$ gauge symmetry is broken down to the Standard Model one, in which case the scalar potential that follows from Eq. (1) gives $\langle\Sigma\rangle \sim M_{75}/\lambda_{75}$. The $\bar{\theta}\Sigma h$, $\theta\Sigma\bar{h}$, and $\theta\bar{\theta}$ terms in W effect the doublet-triplet mechanism via the mass matrix

$$\begin{array}{c c} & \begin{array}{cc} \bar{h}_3 & \bar{\theta}_3 \end{array} \\ \begin{array}{c} h_3 \\ \theta_3 \end{array} & \begin{pmatrix} 0 & \lambda_4\langle\Sigma\rangle \\ \lambda_5\langle\Sigma\rangle & M_{50} \end{pmatrix} \end{array}\ , \quad (2)$$

where the subscript '3' indicates the $SU(2)_L$ singlet, $SU(3)_C$ triplet component of the corresponding $SU(5)$ representation. This matrix clearly yields massive ($\sim \langle\Sigma\rangle \sim M_{\rm GUT}$) Higgs triplets ($h_3, \bar{h}_3$), whereas the doublets ($h_2, \bar{h}_2$) remain massless. The M_{50} term is not required for a successful doublet-triplet splitting. However, it is introduced in order to give large masses to the many leftover components of the **50**,$\overline{\mathbf{50}}$ representations. The last two terms in W (1) provide the Yukawa matrices for the Standard Model fermions, implying the usual relations (*e.g.*, $\lambda_b = \lambda_\tau$).

Despite the above natural solution to the doublet-triplet splitting problem, the magnitude of the dimension-five ($d = 5$) proton decay operators still needs to be assessed. The crucial element in this calculation is the effective $h_3\bar{h}_3$ mixing term. If in Eq. (1) M_{50} were allowed to vanish, then there would be no mixing whatever, and the $d = 5$ operators would be negligible. In practice M_{50} cannot vanish, and we are left with two possibilities: (i) $M_{50} \sim \langle\Sigma\rangle$, and (ii) $M_{50} \gg \langle\Sigma\rangle$. The first case implies an effective Higgs-triplet mixing term of the same magnitude as in the minimal $SU(5)$ model, and therefore similar difficulties in suppressing proton decay. However, this case is not really viable since above the GUT scale the large **50**,$\overline{\mathbf{50}}$ representations increase the $SU(5)$ beta function so much that the gauge coupling becomes non-perturbative before reaching the Planck scale [5, 15]. We are left with the second alternative with $M_{50} \sim M$, where $M = M_{Pl}/\sqrt{8\pi} \approx 10^{18}\,\mathrm{GeV}$ is the appropriate gravitational scale. Unfortunately, this choice leads to a see-saw type mass for the Higgs triplets: $m_{h_3,\bar{h}_3} \sim \langle\Sigma\rangle^2/M \sim 10^{14}\,\mathrm{GeV}$, and effective mixing of the same magnitude, which makes proton decay much too fast.

Various variants of the MDM have been proposed to deal with the proton decay problem in a more effective way [10, 14, 15]. Here we follow the suggestion in Ref. [15], whereby the following additional fields are introduced: θ' (**50**), $\bar{\theta}'$ ($\overline{\mathbf{50}}$), h' (**5**), $\bar{h}'$ ($\overline{\mathbf{5}}$). The superpotential for the model is that in Eq. (1) with $M_{50} \equiv 0$, and supplemented by

$$W' = \lambda_4'\,\bar{\theta}'\Sigma h' + \lambda_5'\,\theta'\Sigma\bar{h}' + M_{50}'\,\theta\bar{\theta}' + M_{50}'\,\theta'\bar{\theta}\ , \quad (3)$$

where we again take $M'_{50} \sim M$. These interactions lead to the following generalized Higgs-triplet mass matrix

$$\begin{array}{c} \\ h_3 \\ \theta'_3 \\ h'_3 \\ \theta_3 \end{array} \begin{array}{c} \begin{array}{cccc} \bar{h}'_3 & \bar{\theta}_3 & \bar{h}_3 & \bar{\theta}'_3 \end{array} \\ \begin{pmatrix} 0 & \lambda_4\langle\Sigma\rangle & 0 & 0 \\ \lambda'_5\langle\Sigma\rangle & M'_{50} & 0 & 0 \\ 0 & 0 & 0 & \lambda'_4\langle\Sigma\rangle \\ 0 & 0 & \lambda_5\langle\Sigma\rangle & M'_{50} \end{pmatrix} \end{array} , \tag{4}$$

and effective interactions [15]

$$\left(\lambda_4\lambda'_5\frac{\langle\Sigma\rangle^2}{M'_{50}}\right) h_3\bar{h}'_3 + \left(\lambda'_4\lambda_5\frac{\langle\Sigma\rangle^2}{M'_{50}}\right) h'_3\bar{h}_3 \equiv M_{H_3}\, h_3\bar{h}'_3 + M_{\bar{H}_3}\, h'_3\bar{h}_3 \ . \tag{5}$$

Since there is no effective interaction between h_3 and $\bar{h}_3$ (the only triplets that interact with the Standard Model fermions), the $d = 5$ operator is negligible.

If the superpotential $W+W'$ were the complete model, we would have managed to make all the non-minimal fields sufficiently heavy or non-interacting. However, we would have left two pairs of Higgs doublets $h_2, \bar{h}_2$ and $h'_2, \bar{h}'_2$ with no apparent use for the second pair, and if light, with severe trouble with gauge coupling unification. Let us assume the existence of a mass term $M_{h'}h'\bar{h}'$, with no specific origin for $M_{h'}$ for now. Such a term contains $M_{h'}h'_3\bar{h}'_3$, which "hooks up" the two disconnected pieces in Eq. (5) and allows $d = 5$ proton decay to occur, with an operator proportional to

$$\frac{1}{M_{H_{\rm eff}}} \equiv \frac{M_{h'}}{M_{H_3}M_{\bar{H}_3}} \sim \frac{M_{h'}}{[\langle\Sigma\rangle^2/M'_{50}]^2} \ , \tag{6}$$

where $M_{H_{\rm eff}}$ is the effective Higgs triplet mass. Since in the minimal $SU(5)$ model with $M_{H_{\rm eff}} = M_{H_3} \gtrsim 10^{17}$ GeV, the present experimental bounds on proton decay are satisfied without strong restrictions on the parameter space [16, 17], we effectively require $M_{h'} \lesssim 10^{11}$ GeV.

But where does this intermediate scale come from? It has been suggested that this scale could be generated dynamically via the breaking of a Peccei-Quinn symmetry [35, 15]. A more modern and economical approach to the generation of intermediate scales, especially in the context of supergravity, is to consider the condensation of a hidden sector gauge group that triggers supersymmetry breaking. Non-renormalizable interactions coupling hidden sector matter fields to observable fields may then naturally generate the intermediate scale.[2] Specifically, we add to our model the following superpotential terms[3]

$$W'' = \lambda_7\, h\bar{h}\,\frac{(T\bar{T})^2}{M^3} + \lambda'_7\, h'\bar{h}'\,\frac{T\bar{T}}{M} \ , \tag{7}$$

[2]This mechanism is commonly available in string model building [36].

[3]The apparent asymmetry between the $h\bar{h}$ and $h'\bar{h}'$ couplings may be understood on the basis of additional local $U(1)$ quantum numbers, which are broken near the Planck scale and are carried by both hidden and observable sector particles, as is common in string model building [36]. For further symmetry arguments motivating these choices, see *e.g.*, Ref. [37].

where $T\bar{T}$ is a gauge-singlet hidden-sector composite (*e.g.*, $\mathbf{4\bar{4}}$ in $SU(4)$). When the hidden sector condenses, we generate dynamically two mass scales:

$$M_{h'} = \lambda_7' \frac{\langle T\bar{T}\rangle}{M} , \qquad \mu = \lambda_7 \frac{\langle T\bar{T}\rangle^2}{M^3} . \tag{8}$$

Note that for $\langle T\bar{T}\rangle/M \sim 10^{10}\,\mathrm{GeV}$, we would obtain for the masses of the extra pair of doublets $M_{h'} \sim 10^{10}\,\mathrm{GeV}$, and an effective Higgs-triplet mixing which satisfies proton decay constraints automatically. We would also obtain dynamically[4] a very desirable Higgs mixing parameter $\mu \sim 100\,\mathrm{GeV}$. In the next section we explore the hidden sector of the model with these phenomenological constraints in mind.

One of the main model-building shortcomings of $SU(5)$ GUTs is the not-so-obvious source of neutrino masses. In fact, neutrino masses can be introduced by simply adding right-handed neutrino ($SU(5)$ singlet) fields to the model. To implement the standard see-saw mechanism we introduce three singlet fields $\nu^c_{1,2,3}$ with the following superpotential

$$W''' = \lambda_3^{ij}\, \bar{f}_i \nu^c_j h + \lambda_6^{ij}\, \nu^c_i \nu^c_j \frac{T\bar{T}}{M} . \tag{9}$$

After hidden sector condensation and electroweak symmetry breaking, we obtain the following see-saw neutrino mass matrix

$$\begin{array}{cc} & \begin{array}{cc} \nu_j & \qquad \nu^c_j \end{array} \\ \begin{array}{c} \nu_i \\ \nu^c_i \end{array} & \begin{pmatrix} 0 & \lambda_3^{ij} v_2 \\ \lambda_3^{ji} v_2 & \lambda_6^{ij} \langle T\bar{T}\rangle/M \end{pmatrix} \end{array} , \tag{10}$$

and light neutrino see-saw masses $m_\nu \sim \lambda_2^3 v_2^2/[\lambda_6\langle T\bar{T}\rangle/M]$. For simplicity, in what follows we assume $\lambda_6^{ij} = \lambda_6^i \delta_{ij}$. With our above desired value of $\langle T\bar{T}\rangle/M \sim M_{\nu^c} \sim 10^{10}\,\mathrm{GeV}$, and $\lambda_3 v_2 \sim 10\,\mathrm{GeV}$, typical see-saw light neutrino masses follow, *i.e.*, $m_{\nu_\tau} \sim 10\,\mathrm{eV}$. Further discussion of the consequences of this see-saw matrix for the light neutrino masses and mixing angles is given in Sec. 5 below.

3 The hidden sector

Our supergravity model is endowed with a hidden sector which communicates with the observable sector via gravitational interactions (or via $U(1)$ gauge interactions broken near the Planck scale). The hidden sector consists of a hidden gauge group and a set of matter representations, which for convenience we take to be $SU(N_c)$ with N_f flavors ($T_i, \bar{T}_i,\ i = 1 \rightarrow N_f$) and $N_f < N_c$. This gauge group starts with a gauge coupling g at the Planck scale ($Q = M$), and becomes strongly interacting at the condensation scale defined by

$$\Lambda = M e^{8\pi^2/\beta g^2} , \tag{11}$$

[4]This dynamical generation of the μ parameter via non-renormalizable interactions is also familiar from string model-building [36, 38, 37].

where the beta function is given by $\beta = -3N_c + N_f$. For simplicity we assume that all the flavors are "light", *i.e.*, they have masses[5] $m \ll \Lambda$. At the condensation scale, the strongly interacting theory is described in terms of composite "meson" fields $T_i\bar{T}_i$. The dynamics of this system can be obtained from an effective Lagrangian with the following non-perturbative superpotential [40]

$$W_{\rm non-pert} = (N_c - N_f)\, \frac{\Lambda^{(3N_c-N_f)/(N_c-N_f)}}{(\det T\bar{T})^{1/(N_c-N_f)}} + {\rm Tr}\,(mT\bar{T}) \ . \tag{12}$$

Minimization of the corresponding scalar potential results in the following expectation values for the mesons fields $\langle T\bar{T}\rangle$ (we work in a diagonal flavor basis)

$$\langle T\bar{T}\rangle = \Lambda^{(3N_c-N_f)/N_c}\,(\det m)^{1/N_c}\, m^{-1} = \Lambda^3 \left(\frac{m}{\Lambda}\right)^{N_f/N_c} \frac{1}{m} = \Lambda^2\, x^{(N_f/N_c)-1} \ , \tag{13}$$

where in the last expression we have defined $x \equiv m/\Lambda$, with $x < 1$. Inserting this expectation value in $W_{\rm non-pert}$, we obtain

$$\langle W\rangle = N_c\,\Lambda^3\, x^{N_f/N_c} \ , \tag{14}$$

where W includes all perturbative and non-perturbative contributions. In a supergravity theory, the scale of supersymmetry breaking is determined by the gravitino mass: $m_{3/2} = \langle e^K\, W\rangle$, where K is the Kähler potential. In simple models $K = \sum \phi_i\phi_i^\dagger$, and thus $\langle K\rangle = 0$. More complicated forms of K are obtained in string models (where the dilaton and moduli fields play an important role). For our present purposes, we simply assume that $\langle e^K\rangle \sim 1$. This implies that $\langle W\rangle$ is the sole source of supersymmetry breaking, *i.e.*,

$$m_{3/2} \sim \langle W\rangle \sim \left(\frac{\Lambda}{M}\right)^3 x^{N_f/N_c}\, M \ , \tag{15}$$

where we have restored the units in the expression for $m_{3/2}$.

With the results in Eqs. (13) and (15) for $\langle T\bar{T}\rangle$ and $m_{3/2}$, we can now investigate the conditions on N_c, N_f, and x that would yield the desired results: $\langle T\bar{T}\rangle/M = 10^p\,$GeV and $m_{3/2} = 10^q\,$GeV, with $p \sim 10$ and $q \sim 3$. In terms of p and q, we can solve simultaneously Eqs. (13), (15), and (11), to obtain

$$N_c = \frac{p-q}{18-q}\, N_f + \frac{8\pi^2}{g^2}\, \frac{\log_{10} e}{18-q} \ , \tag{16}$$

and

$$x = 10^{2N_c\,(\frac{3}{2}p-q-9)/\beta} \ . \tag{17}$$

Thus, for a given value of g and N_f, we obtain N_c (and thus β) from Eq. (16). With this value of N_c, x is determined from Eq. (17), and Λ from Eq. (11). For the desired

[5] Massless flavors lead to pathologies (*i.e.*, no vacuum), which can nonetheless be remedied by invoking supersymmetry-breaking masses for the $T_i, \bar{T}_i$ fields [39].

$p = 10$ and $q = 3$, and with the sensible inputs $N_f = 1$ and $g = 0.7$, we obtain $N_c = 5$, $x \approx 0.01$, and $\Lambda \approx 10^{13}\,\mathrm{GeV}$. That is, an $SU(5)$ hidden gauge group with one light flavor. The general constraints on N_c and N_f for given values of g are shown in Fig. 1, for $q = 2 \to 3$ (*i.e.*, $m_{3/2} = 100\,\mathrm{GeV} \to 1\,\mathrm{TeV}$) and $p = 10$.

We do not address here the calculation of the observable-sector soft-supersymmetry-breaking scalar and gaugino masses, since these depend on the specific choices for the Kähler function and the gauge kinetic function, although their overall scale is already determined by $m_{3/2}$. The "flat" choice $K = \sum \phi_i \phi_j^\dagger$ leads to the usual universal scalar masses, but this choice is not unique.

4 Unification and proton decay

The revamped MDM presented in the two previous sections contains several departures from conventional gauge coupling unification: (i) there is a pair of Higgs doublets with intermediate-scale masses ($M_{h'} \sim 10^{10}\,\mathrm{GeV}$), (ii) there is a richer structure of GUT particles, including two pairs of Higgs triplets (from the $\mathbf{5}$,$\overline{\mathbf{5}}$ representations) with masses $M_{H_3,\bar{H}_3} \sim 10^{14}\,\mathrm{GeV}$, and (iii) there is a spectrum of masses for the different components of the $\mathbf{75}$, all near the unification scale. There is also a hidden gauge group, with an in-principle independent gauge coupling at the Planck scale (denoted by g in Sec. 3).[6] Fortunately, the issue of gauge coupling unification in the observable sector has already been addressed in detail in Ref. [15]. Those calculations are directly applicable to our model since the observable matter content and spectrum of the two models is the same, even though the dynamics providing the intermediate scale are different. Thus, here we limit ourselves to a brief summary of the relevant issues.

Writing down the one-loop RGEs for the gauge couplings, including a common supersymmetric threshold at $M_{\rm SUSY}$, one can eliminate the unified $SU(5)$ coupling and obtain the following two relations [15, 41]

$$\left(3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1}\right)(M_Z) = \frac{1}{2\pi}\left\{\frac{12}{5}\ln\frac{M_{H_3}M_{\bar{H}_3}}{M_{h'}M_Z} - 2\ln\frac{M_{\rm SUSY}}{M_Z} - 23.3\right\} \quad (18)$$

$$\left(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1}\right)(M_Z) = \frac{1}{2\pi}\left\{36\ln\frac{(M_V^2 M_\Sigma)^{1/3}}{M_Z} + 8\ln\frac{M_{\rm SUSY}}{M_Z} + 12.1\right\} \quad (19)$$

In these relations, $M_V = 3\sqrt{15}(g_5/\lambda_{75})M_{75}$ is the mass of the GUT gauge bosons, and the explicit constants come from the splittings of the $\mathbf{75}$ relative to the $M_\Sigma = 5M_{75}$ mass of its ($\mathbf{8}$,$\mathbf{3}$) component. The above relations can be made more accurate by the inclusion of realistic low-energy supersymmetric thresholds, two-loop gauge coupling RGEs, and smooth decoupling of heavy particles. Once this is done, and the latest values of the Standard Model gauge couplings are input (*i.e.*, $\alpha^{-1} = 127.9 \pm 0.2$,

[6] In the spirit of string unified models one could assume that the observable and hidden gauge couplings are related at the Planck scale or at the string scale ($M_{\rm str} \approx 4 \times 10^{17}\,\mathrm{GeV}$).

$\sin^2\theta_W = 0.2314 \pm 0.0004$, $\alpha_3 = 0.118 \pm 0.007$), one obtains the following 1σ allowed intervals [15]:

$$1.4 \times 10^{17}\,\mathrm{GeV} \le \quad \frac{M_{H_3} M_{\bar{H}_3}}{M_{h'}} \quad \le 5.5 \times 10^{20}\,\mathrm{GeV}\ , \tag{20}$$

$$8.4 \times 10^{15}\,\mathrm{GeV} \le \quad (M_V^2 M_\Sigma)^{1/3} \quad \le 2.6 \times 10^{16}\,\mathrm{GeV}\ . \tag{21}$$

It is evident that our choices above, *i.e.*, $M_{H_3} \sim M_{\bar{H}_3} \sim 10^{14}\,\mathrm{GeV}$ and $M_{h'} \sim 10^{10}\,\mathrm{GeV}$, are perfectly consistent with the constraint in Eq. (20). The same is true for the middle-of-the-road choice $M_V \sim M_\Sigma$ (*i.e.*, $\lambda_{75} \sim g_5$), which yields a GUT scale close to $10^{16}\,\mathrm{GeV}$.

We recall that we have set the masses of the $\mathbf{50},\overline{\mathbf{50}}$ representations at the gravitational scale $M \approx 10^{18}\,\mathrm{GeV}$ in order to prevent the onset of a non-perturbative $SU(5)$ regime below the Planck scale. Nonetheless, because of the needed GUT-scale **75** representation, the unified gauge coupling grows above the unification scale. However, it has been demonstrated that this coupling remains in the perturbative regime, *i.e.*, $\alpha \lesssim 0.1$ [15]. One could assume that the corresponding gauge coupling at the gravitational scale ($g \approx 1$) is related to the gauge coupling from the hidden sector gauge group discussed in Sec. 3, as would be the case in string models. This relation would help to further constrain the viable hidden sector choices. For instance, assuming a "super-unified" situation, where hidden and observable gauge couplings are equal near the gravitational scale, the constraints on the hidden sector choice can be read off Fig. 1 ($g = 1$ curves).

Concerning proton decay, gauge-boson-mediated dimension-six operators depend on $1/M_V^2$. From Eq. (21), M_V is not expected to be much below $10^{16}\,\mathrm{GeV}$, unless $\lambda_{75} \gg g_5$, but this case is unlikely since λ_{75} would be in the non-perturbative regime. Thus, we don't expect a particular enhancement of dimension-six operators in this model. More interesting is the situation with the dimension-five proton decay operators, which depend on the effective Higgs triplet mass ($M_{H_{\rm eff}}$) defined in Eq. (6). The dominant proton partial lifetime is given by [17, 19]

$$\tau(p \to K^+\bar{\nu}_\mu) = 2.0 \times 10^{31}\,\mathrm{y} \left| \frac{0.0056\,\mathrm{GeV}^3}{\beta}\, \frac{0.67}{A_S}\, \frac{\sin 2\beta}{1 + y^{tK}}\, \frac{M_{H_{\rm eff}}}{10^{17}\,\mathrm{GeV}}\, \frac{\mathrm{TeV}^{-1}}{f} \right|^2\ , \tag{22}$$

where $\beta = (5.6 \pm 0.8) \times 10^{-3}\,\mathrm{GeV}^3$ is the relevant hadronic matrix element, A_S is the short-distance renormalization factor, and y^{tK} parametrizes the contribution of the third family relative to the first two ($|y^{tK}| \approx 2$ for $m_t = 175\,\mathrm{GeV}$) with an undetermined phase. The f functions are the one-loop integrals which behave as $1/f \approx m_{\tilde{q}}^2/m_{\widetilde{W}}$ for $m_{\tilde{q}} \gg m_{\widetilde{W}}$.

From unification constraints, Eq. (20) indicates that $M_{H_{\rm eff}} > 1.4 \times 10^{17}\,\mathrm{GeV} \approx 10 M_V$. In this case, Eq. (22) and Ref. [16] show that the present Kamiokande limit $\tau(p \to \bar{\nu}K^+) > 1 \times 10^{32}\,\mathrm{y}$ [42], is satisfied provided $\tan\beta$ is not too large ($\tan\beta \lesssim 5$) and the universal scalar mass $m_0 > 300\,\mathrm{GeV}$. On the other hand, in our model we obtain $M_{H_{\rm eff}} \gtrsim 10^{18}\,\mathrm{GeV} \approx 100 M_V$, and the experimental limit is satisfied rather comfortably, even for large values of $\tan\beta$ and presently accessible supersymmetric particle masses. For instance, for $m_{\tilde{q}} \approx 300\,(600)\,\mathrm{GeV}$ and $m_{\widetilde{W}} \approx 80\,\mathrm{GeV}$, $\tan\beta \lesssim 10\,(40)$

is required. Thus, $p \to \bar{\nu} K^+$ remains the dominant mode for proton decay, with good prospects for observation at the upcoming SuperKamiokande experiment and the proposed ICARUS facility. Note that the much-weakened proton-decay upper-bound on $\tan\beta$ offers a new possibility in the study of Yukawa coupling unification in $SU(5)$ GUTs (*i.e.*, $\lambda_b = \lambda_\tau$), which now also allow the so-called "large-$\tan\beta$" solution [43].

5 Cosmic baryon asymmetry

With the realization of significant electroweak baryon number violation at high temperatures, which occurs through $(B+L)$-violating but $(B–L)$-conserving non-perturbative sphaleron interactions [44], several new mechanisms for generating the cosmic baryon asymmetry have been proposed [45]. These mechanisms produce a primordial lepton asymmetry (leptogenesis), which is then recycled by sphaleron interactions into a baryon asymmetry at the electroweak scale. It is important to note that primordial $(B–L)$-conserving asymmetries, such as those produced in traditional $SU(5)$ GUT baryogenesis, are likely to be wiped out by the sphaleron interactions [46]. Therefore, in the context of $SU(5)$ GUTs, the leptogenesis-based mechanisms may be unavoidable. Here we consider the simplest of these mechanisms, based on the out-of-equilibrium decay of right-handed neutrinos, as first suggested in Ref. [47],[7] and extended to supersymmetry in Refs. [48, 49], and to $SU(5) \times U(1)$ GUTs in Refs. [50, 51]. We note that the lepton-asymmetric decays of right-handed sneutrino condensates [52, 53], may provide an additional contribution to the lepton asymmetry that we discuss below.

In order to satisfy the out-of-equilibrium condition in the decay of the right-handed neutrinos, one could follow the standard procedure of demanding that the $\nu^c_{1,2,3}$ decay rate be less than the expansion rate of the Universe at the time of ν^c decay. This condition leads to constraints on the λ_3 couplings of the right-handed neutrinos, that can be undesirable when trying to use the same couplings to compute the corresponding light see-saw neutrino masses. Even more problematic can be the need to obtain the surviving lepton asymmetry solely from the decays of the lightest right-handed neutrino (ν^c_1), since the asymmetry produced in the decays of $\nu^c_{2,3}$ is typically wiped out by the ν^c_1 interactions. Such potential difficulties have been exemplified in Ref. [51]. Instead, here we follow an alternative scenario [54],[8] whereby the right-handed neutrinos are produced in the decays of the inflaton subsequent to inflation. The COBE data on the anisotropy of the cosmic microwave brackground radiation, interpreted in the context of inflation, allows one to deduce the inflaton mass to be $m_\eta \sim 10^{11}\,\mathrm{GeV}$ and the reheating temperature $T_R \sim 10^8\,\mathrm{GeV}$ [48]. The ν^c then decay immediately after inflation and out of equilibrium at the temperature

[7]Before the realization of the importance of the sphaleron interactions, Ref. [14] pointed out the possibility of generating a baryon asymmetry in the decay of right-handed neutrinos via baryon number violating GUT interactions.

[8]Below we show that in our model, the traditional out-of-equilibrium scenario is also viable.

$T_R \ll M_{\nu^c}$, *as long as* $M_{\nu_c} < m_\eta \sim 10^{11}\,\mathrm{GeV}$. Interestingly, the constraint from proton decay (see Sec. 2) ensures that this condition is satisfied *automatically*.

The primordial lepton asymmetry, when reprocessed by sphaleron interactions, leads to a similar baryon asymmetry [48]

$$\frac{n_B}{n_\gamma} \sim \frac{n_L}{n_\gamma} \sim \left(\frac{m_\eta}{M_{Pl}}\right)^{1/2} \epsilon \sim 10^{-4}\ \epsilon\ , \tag{23}$$

where the asymmetry parameter (ϵ) due to the decay of the ith-generation neutrino and sneutrino is given by [48]

$$\epsilon_i = \frac{1}{2\pi(\lambda_3^\dagger\lambda_3)_{ii}} \sum_j \left(\mathrm{Im}\,[(\lambda_3^\dagger\lambda_3)_{ij}]^2\right)\ g(M^2_{\nu^c_j}/M^2_{\nu^c_i})\ , \tag{24}$$

with

$$g(x) = 4\sqrt{x}\ \ln\frac{1+x}{x}\ . \tag{25}$$

To proceed we need to manipulate the entries in λ_3, which has remained as yet unspecified. We define the unitary rotation matrix U, such that $\hat{\lambda}_3 = U\lambda_3U^\dagger$, where $\hat{\lambda}_3$ is the diagonal matrix of eigenvalues of λ_3. Experience with the quark mixing matrix leads us to assume that U differs little from the identity matrix: $U = \mathbf{1} + R$, with [51]

$$R = \begin{pmatrix} 0 & \theta_{12} & 0 \\ -\theta_{12}^* & 0 & \theta_{23} \\ 0 & -\theta_{23}^* & 0 \end{pmatrix}\ . \tag{26}$$

With this ansatz we obtain to lowest non-vanishing order

$$(\lambda_3^\dagger\lambda_3)_{ii} = |\hat{\lambda}_3^i|^2 + \sum_j |\hat{\lambda}_3^j|^2\,|\theta_{ij}|^2\ , \tag{27}$$

$$(\lambda_3^\dagger\lambda_3)_{ij} = |\theta_{ij}|e^{i\phi_{ij}}\left[|\hat{\lambda}_3^i|^2 - |\hat{\lambda}_3^j|^2\right] \quad (i \neq j)\ , \tag{28}$$

where $\phi_{ij} = \mathrm{Arg}\,[\theta_{ij}]$. Thus, Eq. (24) becomes

$$\epsilon_i = \frac{1}{2\pi}\,\frac{\sum_{j\neq i}|\theta_{ij}|^2\sin 2\phi_{ij}\,[|\hat{\lambda}_3^i|^2 - |\hat{\lambda}_3^j|^2]^2\ g(M^2_{\nu^c_j}/M^2_{\nu^c_i})}{|\hat{\lambda}_3^i|^2 + \sum_j|\hat{\lambda}_3^j|^2\,|\theta_{ij}|^2}\ . \tag{29}$$

Because of the several unknown parameters in the above expressions, and the inherent uncertainties in this type of calculations, we will be content with presenting a plausible scenario leading to interesting lepton asymmetries and see-saw neutrino masses. For simplicity let us assume that the λ_6 matrix is proportional to the unit matrix, *i.e.*,

$$M_{\nu_1^c} = M_{\nu_2^c} = M_{\nu_3^c} = M_{\nu^c} = \lambda_6\,\frac{\langle T\bar{T}\rangle}{M}\ . \tag{30}$$

The light neutrino mass matrix then becomes $M_\nu = \lambda_3\lambda_3^T v_2^2/M_{\nu^c}$. If we neglect the CP violating phases (a not necessarily justified approximation), the matrix U which diagonalizes $\lambda_3\lambda_3^\dagger$, also diagonalizes $\lambda_3\lambda_3^T$ and the physical neutrino masses become (up renormalization group scaling corrections [55])

$$m_{\nu_i} \approx \frac{(\hat\lambda_3^i v_2)^2}{M_{\nu^c}} \ . \tag{31}$$

Furthermore, in our ansatz the (small) neutrino mixing angles are given by $\theta_{e\mu} = \theta_{12}$, $\theta_{e\tau} = 0$, and $\theta_{\mu\tau} = \theta_{23}$. As we will see shortly, these mixing angles are unrestricted from lepton asymmetry considerations, and thus could accomodate the MSW solution to the solar neutrino problem ($\nu_e \leftrightarrow \nu_\mu$) and lead to interesting $\nu_\mu \leftrightarrow \nu_\tau$ oscillations at the CHORUS and NOMAD, and P803 experiments at CERN and Fermilab respectively.

From Eq. (31) we see that $m_{\nu_\tau} \approx (\hat\lambda_3^3 v_2)^2/M_{\nu^c} = [\hat\lambda_3^3 \sin\beta(174\,\mathrm{GeV})]^2/M_{\nu^c}$. With $M_{\nu^c} \sim 10^{10}\,\mathrm{GeV}$ and $\hat\lambda_3^3 \approx 0.1$, we get $m_{\nu_\tau} \sim 15\,(30)\,\mathrm{eV}$ for $\tan\beta \sim 1\,(\tan\beta \gg 1)$. This range of tau neutrino masses provide an adequate and desirable hot dark matter component of the Universe. Thus, in what follows we take $\hat\lambda_3^3 = 0.1$. It is also natural to assume that the remaining eigenvalues of the λ_3 matrix are hierarchically smaller, *i.e.*, $\hat\lambda_3^1 \ll \hat\lambda_3^2 \ll \hat\lambda_3^3$. For instance, $\hat\lambda_3^2 \sim \frac{1}{100}\hat\lambda_3^3$ yields $m_{\nu_\mu} \sim 10^{-3}\,\mathrm{eV}$, consistent with solutions to the solar neutrino problem via the MSW mechanism. (These hierarchies are comparable to those in the up-quark Yukawa matrix.)

Going back to the calculation of the lepton asymmetries, with our hierarchical assumption for $\hat\lambda_3$, from Eq. (29) we obtain

$$\begin{aligned}
\epsilon_1 &\approx \frac{2\ln 2}{\pi}\left(\hat\lambda_3^2\right)^2 \sin 2\phi_{12} \sim 10^{-6}\,\phi_{12} \ , && (32)\\
\epsilon_2 &\approx \frac{2\ln 2}{\pi}\left(\hat\lambda_3^3\right)^2 \sin 2\phi_{23} \sim 10^{-2}\,\phi_{23} \ , && (33)\\
\epsilon_3 &\approx \frac{2\ln 2}{\pi}\left(\hat\lambda_3^3\right)^2 |\theta_{23}|^2 \sin 2\phi_{23} \sim 10^{-2}\,|\theta_{23}|^2\phi_{23} \ . && (34)
\end{aligned}$$

With the expression for the estimated baryon asymmetry in Eq. (23), we would get the desired result of few $\times\, 10^{-10}$ for $\phi_{12} \sim 1$ and $\phi_{23} \ll 1$. The natural choice would be maximal CP violation in the θ_{12} entry in the rotation matrix R (see Eq. (26)) and no CP violation elsewhere in the matrix (unless new entropy diluting sources are introduced to reduce $\epsilon_1 + \epsilon_2 + \epsilon_3$). These results would be affected somewhat if one allows a non-trivial structure to the matrix λ_6 (*i.e.*, relaxing the assumption in Eq. (30)).

We now remark that this model is also viable in the traditional out-of-equilibrium scenario, where ϵ_1 is the only surviving asymmetry. The out-of-equilibrium condition at $T = M_{\nu_1^c} \sim 10^{10}\,\mathrm{GeV}$,

$$\Gamma_{\nu_1^c} = \frac{(\lambda_3^\dagger\lambda_3)_{11}}{16\pi} M_{\nu_1^c} < 1.66 g_*^{1/2}\frac{T^2}{M_{Pl}} = H \ , \tag{35}$$

is satisfied for (using Eq. (27))

$$(\lambda_3^\dagger \lambda_3)_{11} = |\hat{\lambda}_3^1|^2 + |\hat{\lambda}_3^2|^2 \, |\theta_{12}|^2 \lesssim 10^{-6} \ , \tag{36}$$

which is consistent with our hierarchical assumption. However, in this case the calculation of the leptonic asymmetry has a larger ($\sim 10^{-2}$ [48]) coefficient than in Eq. (23), requiring a non-maximal CP violating phase $\phi_{12} \sim 10^{-2}$.

Finally, let us comment on whether or not the sphaleron interactions may wash away the leptonic asymmetry produced above. This could in principle happen if the non-renormalizable operators obtained when integrating out the right-handed neutrino fields, *i.e.*, $(\lambda_3/M_{\nu^c})LLHH$, where L is the lepton doublet in $\bar{f}$ and H the Higgs doublet in h, are in equilibrium with the sphaleron interactions [47]. It has been shown [56] that to prevent the erasure of the asymmetry, one must demand $M_{\nu^c} \gtrsim (\lambda_3)^2 \, 3 \times 10^9 \, \mathrm{GeV}$, which is always satisfied for our choices of λ_3 and M_{ν^c}.

6 Comparison with $SU(5) \times U(1)$

The revamped MDM presented in the previous sections has several appealing phenomenological features, constituting an interesting example of traditional grand unified model building. Nonetheless, it is apparent that the model is rather non-minimal or uneconomical. For instance, a **75** needs to be used for GUT symmetry breaking, greatly increasing the size of the GUT particle spectrum. Moreover, the $\mathbf{50},\overline{\mathbf{50}}$ to effect the doublet-triplet splitting problem make the unified gauge coupling so large above the GUT scale that they need to be taken at the gravitational scale. The doublet-triplet splitting is tamed, but proton decay can still be too fast because of the "useless" pieces of the $\mathbf{50},\overline{\mathbf{50}}$ representations which need to be made heavy, resulting in the otherwise-not-needed doubling of these representations and of the Higgs pentaplets. Regarding the right-handed neutrinos, their (ad-hoc) introduction has various desirable consequences. However, the Yukawa matrix coupling them to the lepton doublets is arbitrary, with no particular motivation for its desired hierarchical structure.

It is interesting to note that the above critique of the revamped MDM can be circumvented altogether if one extends the gauge group from $SU(5)$ to $SU(5) \times U(1)$ [57, 58, 59]. Gauge symmetry breaking down to the Standard Model gauge group occurs via vacuum expectation values of the H ($\mathbf{10}$) and $\bar{H}$ ($\overline{\mathbf{10}}$) Higgs representations. This is possible because of the "flipping" $u \leftrightarrow d$, $u^c \leftrightarrow d^c$, $e \leftrightarrow \nu$, $e^c \leftrightarrow \nu^c$ involved in the assignment of the Standard Model particles to the $\bar{f} = \{u^c, L\}$ ($\bar{\mathbf{5}}$) and $F = \{Q, d^c, \nu^c\}$ ($\mathbf{10}$) representations. Thus, H and $\bar{H}$ contain one pair of neutral fields $\nu^c_H, \nu^c_{\bar{H}}$, which get vevs along the flat direction $\langle \nu^c_H \rangle = \langle \nu^c_{\bar{H}} \rangle$. There is no need for large GUT representations for symmetry breaking. As is well known (and we review below), this property takes on a much larger magnitude when one attempts to derive these models in string model building.

The missing-partner mechanism, which above involved the couplings $\bar{\theta}\Sigma h$ $[(\overline{\mathbf{50}})(\mathbf{75})(\mathbf{5})]$ and $\theta\Sigma\bar{h}$ $[(\mathbf{50})(\mathbf{75})(\bar{\mathbf{5}})]$, is now effected by the couplings HHh $[(\mathbf{10})(\mathbf{10})(\mathbf{5})]$

and $\bar{H}\bar{H}\bar{h}$ [$(\overline{\mathbf{10}})(\overline{\mathbf{10}})(\overline{\mathbf{5}})$]. First note that no additional representations are needed besides the GUT-breaking Higgs ones. Moreover, the resulting Higgs triplet matrix

$$\begin{array}{cc} & \begin{array}{cc} \bar{h}_3 & \quad d^c_H \end{array} \\ \begin{array}{c} h_3 \\ d^c_{\bar{H}} \end{array} & \left(\begin{array}{cc} 0 & \lambda_4 \langle \nu^c_H \rangle \\ \lambda_5 \langle \nu^c_{\bar{H}} \rangle & 0 \end{array} \right) \end{array} , \tag{37}$$

does not need a large non-zero (22) entry (*c.f.* Eq. (2)) because the "useless" components of the H and $\bar{H}$ representations are eaten by the GUT gauge bosons to become massive or become GUT Higgsinos. This natural zero mass term for $d^c_H d^c_{\bar{H}}$ implies that the dimension-five proton decay operators are negligible. We end up with a very economical GUT Higgs spectrum and no threat of dimension-five operators.

Regarding neutrino masses, the right-handed neutrinos which had to be introduced by hand in the revamped MDM, are now contained in the F ($\mathbf{10}$) representations. Indeed, the coupling $\lambda_3 \bar{f} \nu^c h$ in Eq. (9) is here written as $\lambda_3 \bar{f} e^c h$, with the (unavoidable) right-handed electrons now introduced "by hand". In $SU(5) \times U(1)$ this coupling provides the charged lepton masses. On the other hand, the coupling $\lambda_1 F \bar{f} \bar{h}$, which in Eq. (1) provided the down-quark masses, here provides the up-quark masses and Dirac neutrino masses. (Also, the coupling $\lambda_2 FFh$, which in Eq. (1) provided the up-quark masses, here provides the down-quark masses.) Thus, the right-handed neutrinos are unavoidable in $SU(5) \times U(1)$, and their Yukawa couplings to the lepton doublets are equal to those of the up-quark Yukawa matrix, providing (as discussed in Sec. 5) an automatic and desirable hierarchy in the see-saw neutrino masses. An important distinction between the see-saw mechanism in the revamped MDM and $SU(5) \times U(1)$ is the manner in which the right-handed neutrinos get a mass. In the revamped MDM this is through the superpotential term $\lambda_6 \nu^c \nu^c \langle T\bar{T} \rangle / M$ in Eq. (9), whereas in $SU(5) \times U(1)$ there are two possible sources: (i) through cubic couplings $\lambda_6 F \bar{H} \phi \ni \lambda_6 \langle \nu^c_{\bar{H}} \rangle \nu^c \phi$, where ϕ (with $\langle \phi \rangle = 0$) are $SU(5)$ singlets [58]; and (ii) through non-renormalizable couplings $\lambda_9 FF\bar{H}\bar{H}/M \ni \lambda_9 (\langle \nu^c_{\bar{H}} \rangle^2 / M) \nu^c \nu^c$ [60]. The second form resembles that in the revamped MDM, although the mass scale is likely higher (*i.e.*, $\langle \nu^c_{\bar{H}} \rangle^2 / M \sim 10^{14}\,\mathrm{GeV}$).

These two models also differ somewhat in the calculation of the cosmic baryon asymmetry, besides the possible difference in the right-handed neutrino mass spectrum. Indeed, because the $SU(5) \times U(1)$ gauge symmetry is broken along a flat direction, there is a dilution factor (Δ) in the computation of the lepton asymmetry due to the entropy released by the late-decaying "flaton" field [61, 50]. However, these two effects (ν^c spectrum and Δ) tend to compensate each other and an acceptable baryon asymmetry is typically obtained [50, 51].

There is another cosmological aspect of these models that sets them apart, namely the breaking of the GUT symmetry down to the Standard Model one. In the MDM, $SU(5)$ symmetry breaking via an arbitrary vev of the $\mathbf{75}$ leads to several possible degenerate vacua [62], at least in the context of global supersymmetry. When supergravity effects are taken into account, if the desired vacuum has zero cosmological constant, all the others will be lower in energy, although essentially unreachable

[63]. Thus, if the vev of the **75** can be arranged to be in the desired direction, the Universe will remain in the desired broken phase. In contrast, in $SU(5)\times U(1)$ the breaking down to the Standard Model via the vevs of the $\mathbf{10},\overline{\mathbf{10}}$ along the F- and D-flat direction $\langle\nu^c_H\rangle = \langle\nu^c_{\bar H}\rangle$ is *unique* [58].

Regarding the issue of unification, the revamped MDM requires non-minimal representations to make this possible. In $SU(5)\times U(1)$ traditional grand unification does not occur (although the non-abelian Standard Model gauge groups do unify) and unification is not a test of the model. However, if string unification is desired (at the scale $M_{\rm str}\approx 4\times 10^{17}\,{\rm GeV}$), then non-minimal representations need to be added to the $SU(5)\times U(1)$ model [64].

We have seen that the pair of $\mathbf{50},\overline{\mathbf{50}}$ representations in the revamped MDM need to be put at the gravitational scale. It is then natural to ask whether this model can be obtained from the only known consistent theory of quantum gravity, namely string theory. Because of some technical difficulties which we review below, no attempts have been made to derive the MDM from strings. It is of course well known that $SU(5)\times U(1)$ can be easily derived from strings [65, 66].

The prime constraint in string model-building is that of the massless representations which are allowed when the corresponding gauge group G is represented by a "level-k" Kac-Moody algebra on the world-sheet [33, 67]. The allowed representations must be unitary,

$$\sum_{i=1}^{\rm rank\,G} n_i m_i \le k \ , \tag{38}$$

where n_i are the Dynkin labels of the highest weight representation in question, and m_i are fixed positive integers for a given G. In the case of $SU(n)$: $m_i=1\,,\,\forall i$. In our $SU(5)$ example then $\sum_{i=1}^4 n_i\le k$. Looking up the n_i values, we see that for $k=1$, only $\mathbf{1},\mathbf{5},\overline{\mathbf{5}},\mathbf{10},\overline{\mathbf{10}}$ are unitary. For $k=2$ we find in addition: $\mathbf{15},\mathbf{24},\mathbf{40},\overline{\mathbf{40}},\mathbf{45},\overline{\mathbf{45}},\mathbf{50},\overline{\mathbf{50}},\mathbf{75}$. Only level-one constructions appear to be needed to derive $SU(5)\times U(1)$, whereas at least level-two constructions are required in the MDM. However, this is not the end of the story, since one can also ask whether the allowed representations could possible be massless. This requires calculating the so-called conformal dimension h_r of the representation r,

$$h_r = \frac{C_r}{2k+C_A} \ , \tag{39}$$

where C_r is the Casimir of r, and C_A that of the adjoint representation. If $h_r>1$, the representation is necessarily massive. For $h_r\le 1$ the representation could be massless, although this is not guaranteed since other degrees of freedom may add their own contribution to the conformal dimension making it exceed unity. It is not hard to see that in $SU(5)$ all unitary representations at level one are also massless [33], and thus $SU(5)\times U(1)$ models can be readily constructed at level one. The unitary representations of interest for MDM model-building, which are allowed at level two, have conformal dimensions

$$h_{50,\overline{50}} = \frac{42}{5(k+5)} \ , \qquad h_{75} = \frac{8}{k+5} \ , \tag{40}$$

and are not massless at level two. In fact, $k = 4$ is required to make all these representations massless. Such high-level Kac-Moody constructions have never been attempted.

One intriguing possibility would be to construct level-two $SU(5)$ string models (for recent attempts see Ref. [34]), which should allow the required large MDM representations, although with masses at the Planck scale. Note that this is not necessarily a problem since we already require the $\mathbf{50},\overline{\mathbf{50}}$ to be at that mass scale. If the $\mathbf{75}$ is also raised to that scale, the breaking of $SU(5)$ would occur at the string scale, and this may be difficult to reconcile with gauge coupling unification. It is also worth remarking that in a string model all gauge couplings are related at the string scale, and with $SU(5)$ constructed at level two, the relation would be $\sqrt{2}\, g_5 = g_h$ [68], where g_h is the gauge coupling of the hidden gauge group. Finally, the mass terms in Eqs. (1,3), which would not be allowed if the large MDM representations belonged to the massless spectrum, are expected to arise when they belong to the string massive spectrum. Of course, bridging the gap between the massless and massive spectrum may create problems in obtaining the low-energy effective field theory, but this question cannot be answered until an actual model is constructed along these lines.

7 Conclusions

During the last few years, a great deal of attention has been paid to supersymmetric grand unified theories in light of the precise LEP measurements of the Standard Model gauge couplings. These analyses depend crucially on the details of the low-energy supersymmetric spectrum and the heavy GUT spectrum. Most of the effort to date has been focused on the minimal $SU(5)$ supergravity model, which appears to be running into difficulties regarding unification and proton decay. In addition, there is the nagging doublet-triplet splitting problem that receives no satisfactory explanation. Motivated by these developments, we have reconsidered one of the alternatives to minimal $SU(5)$, where the doublet-triplet splitting is dealt with in a reasonable way via the missing-partner mechanism, and gauge coupling unification is not in jeopardy. We have revamped this model to tame the dimension-five proton decay operators, and to allow see-saw neutrino masses. In order to generate the needed intermediate scale for the right-handed neutrino masses, we have endowed the model with a "modern" hidden sector which can generate dynamically the desired intermediate scale, the scale of supersymmetry breaking, and the Higgs mixing parameter μ. The revamped MDM also provides for the cosmic baryon asymmetry through the Fukugita-Yanagida mechanism via lepton-number-violating decays of the right-handed neutrinos.

We have also contrasted the main features of the revamped MDM against the "flipped" $SU(5)\times U(1)$ model, and basically shown that the former can be considered as a "poor man's" version of the latter. In the realm of string model-building, $SU(5)\times U(1)$ fares rather well, whereas the revamped MDM is very unlikely to be realized, except perhaps if one allows $SU(5)$ symmetry breaking to occur at the string scale.

Acknowledgments

We would like to thank John Ellis and Bruce Campbell for useful discussions. This work has been supported in part by DOE grant DE-FG05-91-ER-40633.

References

[1] J. Ellis, S. Kelley, and D. V. Nanopoulos, Phys. Lett. B **249** (1990) 441 and Phys. Lett. B **260** (1991) 131; P. Langacker and M. Luo, Phys. Rev. D **44** (1991) 817; U. Amaldi, W. de Boer, and H. Fürstenau, Phys. Lett. B **260** (1991) 447; F. Anselmo, L. Cifarelli, A. Peterman, and A. Zichichi, Nuovo Cim. **104A** (1991) 1817. For a review see, *e.g.*, J. L. Lopez, D. V. Nanopoulos, and A. Zichichi, Prog. Part. Nucl. Phys. **33** (1994) 303.

[2] J. Ellis, S. Kelley and D. V. Nanopoulos, Nucl. Phys. B **373** (1992) 55; R. Barbieri and L. Hall, Phys. Rev. Lett. **68** (1992) 752; J. Hisano, H. Murayama, and T. Yanagida, Phys. Rev. Lett. **69** (1992) 1014; K. Hagiwara and Y. Yamada, Phys. Rev. Lett. **70** (1993) 709; F. Anselmo, L. Cifarelli, A. Peterman, and A. Zichichi, Nuovo Cim. **105A** (1992) 1025; P. Langacker and N. Polonsky, Phys. Rev. D **47** (1993) 4028.

[3] A. Chamseddine, R. Arnowitt, and P. Nath, Phys. Rev. Lett. **49** (1982) 970.

[4] M. Bastero-Gil and J. Perez-Mercader, Phys. Lett. B **322** (1994) 355 and LAEFF-95-01 (`hep-ph/9506222`); A. Faraggi and B. Grinstein, Nucl. Phys. B **422** (1994) 3; P. Chankowski, Z. Pluciennik, and S. Pokorski, Nucl. Phys. B **439** (1995) 23.

[5] L. Clavelli and P. Coulter, Phys. Rev. D **51** (1995) 3913 and UAHEP954 (`hep-ph/9507261`).

[6] J. Bagger, K. Matchev, and D. Pierce, Phys. Lett. B **348** (1995) 443.

[7] P. Langacker and N. Polonsky, UPR-0642T (`hep-ph/9503214`).

[8] D. Ring, S. Urano, and R. Arnowitt, CTP-TAMU-01/95 (`hep-ph/9501247`); T. Dasgupta, P. Mamales, and P. Nath (`hep-ph/9501325`).

[9] S. Dimopoulos and H. Georgi, Nucl. Phys. B **193** (1981) 150; N. Sakai, Z. Phys. C**11** (1981) 153.

[10] A. Masiero, D. V. Nanopoulos, K. Tamvakis, and T. Yanagida, Phys. Lett. B **115** (1982) 380; B. Grinstein, Nucl. Phys. B **206** (1982) 387.

[11] E. Witten, Phys. Lett. B **105** (1981) 267; D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B **113** (1982) 151; L. Ibañez and G. Ross, Phys. Lett. B **110** (1982) 215.

[12] K. Inoue, A. Kakuto, and T. Takano, Prog. Theor. Phys. **75** (1986) 664; A. Anselm and A. Johansen, Phys. Lett. B **200** (1988) 331; R. Barbieri, G. Dvali, and A. Strumia, Nucl. Phys. B **391** (1993) 487; R. Barbieri, G. Dvali, and M. Moretti, Phys. Lett. B **312** (1993) 137; R. Barbieri, *et. al.*, Nucl. Phys. B **432** (1994) 491; Z. Berezhiani, C. Csaki, and L. Randall, Nucl. Phys. B **444** (1995) 61; Z. Berezhiani, INFN-FE 02-95 (`hep-ph/9503366`).

[13] J. Polchinski and L. Susskind, Phys. Rev. D **26** (1982) 3661; H. Nilles, M. Srednicki, and D. Wyler, Phys. Lett. B **124** (1982) 237; A. Lahanas, Phys. Lett. B **124** (1982) 341.

[14] A. Masiero, D. V. Nanopoulos, K. Tamvakis, and T. Yanagida, Z. Phys. C **17** (1983) 33.

[15] J. Hisano, T. Moroi, K. Tobe, and T. Yanagida, Phys. Lett. B **342** (1995) 138.

[16] R. Arnowitt and P. Nath, Phys. Rev. Lett. **69** (1992) 725; P. Nath and R. Arnowitt, Phys. Lett. B **287** (1992) 89; R. Arnowitt and P. Nath, Phys. Rev. D **49** (1994) 1479.

[17] J. Hisano, H. Murayama, and T. Yanagida, Nucl. Phys. B **402** (1993) 46.

[18] J. L. Lopez, D. V. Nanopoulos, and A. Zichichi, Phys. Lett. B **291** (1992) 255; J. L. Lopez, D. V. Nanopoulos, and H. Pois, Phys. Rev. D **47** (1993) 2468; R. Arnowitt and P. Nath, Phys. Lett. B **299** (1993) 58 and **307** (1993) 403(E); P. Nath and R. Arnowitt, Phys. Rev. Lett. **70** (1993) 3696; J. L. Lopez, D. V. Nanopoulos, and K. Yuan, Phys. Rev. D **48** (1993) 2766.

[19] J. Hisano, T. Moroi, K. Tobe, and T. Yanagida, TU-470 (`hep-ph/9411298`).

[20] H. Georgi, AIP Conference Proceedings No.23, New York 1975, p. 575; H. Fritzsch and P. Minkowski, Ann. Phys. **93** (1975) 193.

[21] H. Georgi and D. V. Nanopoulos, Nucl. Phys. B **155** (1979) 52 and Nucl. Phys. B **159** (1979) 16.

[22] K. Babu and S. Barr, Phys. Rev. D **48** (1993) 5354, Phys. Rev. D **50** (1994) 3529, BA-95-11 (`hep-ph/9503215`), BA-95-21 (`hep-ph/9506261`).

[23] D. Lee and R. Mohapatra, Phys. Lett. B **324** (1994) 376 and Phys. Rev. D **51** (1995) 1353

[24] G. Anderson, S. Dimopoulos, L. Hall, S. Raby, and G. Starkman, Phys. Rev. D **49** (1994) 3660; L. Hall and S. Raby, Phys. Rev. D **51** (1995) 6524.

[25] K. Babu and S. Barr, Phys. Rev. D **51** (1995) 2463; K. Babu and R. Mohapatra, Phys. Rev. Lett. **74** (1995) 2418; D. Lee and R. Mohapatra, UMD-PP-95-93 (`hep-ph/9502210`); B. Brahmachari and R. Mohapatra, UMD-PP-95-138 (`hep-ph/9505347`).

[26] A. Nelson and L. Randall, Phys. Lett. B **316** (1993) 516.

[27] M. Carena, S. Pokorski, M. Olechowski, and C. Wagner, Nucl. Phys. B **426** (1994) 269; R. Ratazzi, U. Sarid, and L. Hall, Phys. Rev. D **50** (1994) 7048.

[28] F. Borzumati, M. Olechowski, and S. Pokorski, Phys. Lett. B **349** (1995) 311.

[29] M. Olechowski and S. Pokorski, Phys. Lett. B **344** (1995) 201.

[30] J. Ellis and M. Gaillard, Phys. Lett. B **88** (1979) 315; D. V. Nanopoulos and M. Srednicki, Phys. Lett. B **124** (1983) 37.

[31] S. Chaudhuri, S. Chung, and J. Lykken, Fermilab-Pub-94-137-T (`hep-ph/9405374`); G. Cleaver, OHSTPY-HEP-T-94-007 (`hep-th/9409096`).

[32] S. Chaudhuri, S. Chung, G. Hockney, and J. Lykken, Fermilab-Pub-94-413-T (`hep-ph/9501361`); G. Cleaver, OHSTPY-HEP-T-95-003 (`hep-th/9505080`) and OHSTPY-HEP-T-95-004 (`hep-th/9506006`).

[33] J. Ellis, J. L. Lopez, and D. V. Nanopoulos, Phys. Lett. B **245** (1990) 375; A. Font, L. Ibañez, and F. Quevedo, Nucl. Phys. B **345** (1990) 389.

[34] G. Aldazabal, A. Font, L. Ibañez, and A. Uranga, FTUAM-28/94 (`hep-ph/9410206`); G. Aldazabal, (`hep-th/9507162`).

[35] H. Murayama, H. Suzuki, and T. Yanagida, Phys. Lett. B **291** (1992) 418.

[36] See *e.g.*, J. L. Lopez and D. V. Nanopoulos, Phys. Lett. B **251** (1990) 73, Phys. Lett. B **256** (1991) 150, and Phys. Lett. B **268** (1991) 359.

[37] E. Chun, J. Kim, and H. Nilles, Nucl. Phys. B **370** (1992) 105; J. Kim and H. Nilles, Mod. Phys. Lett. A **9** (1994) 3575.

[38] J. Casas and C. Muñoz, Phys. Lett. B **306** (1993) 288.

[39] N. Evans, S. Hsu, and M. Schwetz, YCTP-P8-95 (`hep-th/9503186`); O. Aharony, J. Sonnenschein, M. Peskin, and S. Yankielowicz, SLAC-PUB-6938 (`hep-th/9507013`).

[40] See *e.g.*, N. Seiberg, Phys. Rev. D **49** (1994) 6857 and references therein.

[41] See also, J. Hisano, H. Murayama, and T. Yanagida in Ref. [2] and Ref. [19].

[42] Particle Data Group, Phys. Rev. D **50** (1994) 1173.

[43] S. Kelley, J. L. Lopez, and D. V. Nanopoulos, Phys. Lett. B **274** (1992) 387; V. Barger, M. Berger, and P. Ohmann, Phys. Rev. D **47** (1993) 1093; G. Anderson, S. Dimopoulos, L. Hall, and S. Raby, Phys. Rev. D **47** (1993) 3702.

[44] V. Kuzmin, V. Rubakov, and M. Shaposhnikov, Phys. Lett. B **155** (1985) 36. For a review see, A. Cohen, D. Kaplan, and A. Nelson, Ann. Rev. Nucl. Part. Sci. **43** (1993) 27.

[45] For a recent review see, K. Olive, UMN-TH-1319/94 (`hep-ph/9503342`).

[46] P. Arnold and L. McLerran, Phys. Rev. D **36** (1987) 581 and Phys. Rev. D **37** (1988) 1020.

[47] M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45. See also M. Fukugita and T. Yanagida, Phys. Rev. D **42** (1990) 1285; M. Luty, Phys. Rev. D **45** (1992) 455.

[48] B. Campbell, S. Davidson, and K. Olive, Nucl. Phys. B **399** (1993) 111.

[49] H. Murayama, H. Suzuki, T. Yanagida, J. Yokohama, Phys. Rev. Lett. **70** (1993) 1912 and Phys. Rev. D **50** (1994) 2356.

[50] J. Ellis, D. V. Nanopoulos, and K. Olive, Phys. Lett. B **300** (1993) 121.

[51] J. Ellis, J. L. Lopez, D. V. Nanopoulos, and K. Olive, Phys. Lett. B **308** (1993) 70.

[52] I. Affleck and M. Dine, Nucl. Phys. B **249** (1985) 361. For recent work see, M. Dine, L. Randall, and S. Thomas, Phys. Rev. Lett. **75** (1995) 395 and SLAC-PUB-95-6846 (`hep-ph/9507453`).

[53] B. Campbell, S. Davidson, and K. Olive, Phys. Lett. B **303** (1993) 63; H. Murayama and T. Yanagida, Phys. Lett. B **332** (1994) 349.

[54] A. Dolgov and A. Linde, Phys. Lett. B **116** (1982) 329; D. V. Nanopoulos, K. Olive, and M. Srednicki, Phys. Lett. B **127** (1983) 30.

[55] J. Ellis, J. L. Lopez, and D. V. Nanopoulos, Phys. Lett. B **292** (1992) 189.

[56] J. Cline, K. Kainulainen, and K. Olive, Phys. Rev. Lett. **71** (1993) 2372 and Phys. Rev. D **49** (1994) 6394.

[57] S. Barr, Phys. Lett. B **112** (1982) 219, Phys. Rev. D **40** (1989) 2457; J. Derendinger, J. Kim, and D. V. Nanopoulos, Phys. Lett. B **139** (1984) 170.

[58] I. Antoniadis, J. Ellis, J. Hagelin, and D. V. Nanopoulos, Phys. Lett. B **194** (1987) 231.

[59] For reviews see D. V. Nanopoulos, in "Les Rencontres de Physique de la Vallee d'Aoste: Proceedings: Supernova 1987A, one year later: results and perspectives in particle physics." Edited by M. Greco (Editions Frontieres, 1988), p. 795; J. L. Lopez, Surveys H. E. Phys. **8** (1995) 135 (`hep-ph/9405278`).

[60] I. Antoniadis, J. Ellis, J. Hagelin, and D. V. Nanopoulos, Phys. Lett. B **208** (1988) 209.

[61] B. Campbell, J. Ellis, J. Hagelin, D. V. Nanopoulos, and K. Olive, Phys. Lett. B **197** (1987) 355.

[62] T. Hübsch, S. Meljanac, S. Pallua, and G. Ross, Phys. Lett. B **161** (1985) 122.

[63] S. Weinberg, Phys. Rev. Lett. **48** (1982) 1776.

[64] I. Antoniadis, J. Ellis, S. Kelley, and D. V. Nanopoulos, Phys. Lett. B **272** (1991) 31; S. Kelley, J. L. Lopez, and D. V. Nanopoulos, Phys. Lett. B **278** (1992) 140; D. Bailin and A. Love, Phys. Lett. B **280** (1992) 26; G. Leontaris, Phys. Lett. B **281** (1992) 54; J. L. Lopez, D. V. Nanopoulos, and A. Zichichi, Phys. Rev. D **49** (1994) 343.

[65] I. Antoniadis, J. Ellis, J. Hagelin, and D. V. Nanopoulos, Phys. Lett. B **231** (1989) 65.

[66] J. L. Lopez, D. V. Nanopoulos, and K. Yuan, Nucl. Phys. B **399** (1993) 654.

[67] For a review see, P. Goddard and D. Olive, Int. J. Mod. Phys. A **1** (1986) 303.

[68] P. Ginsparg, Phys. Lett. B **197** (1987) 139.

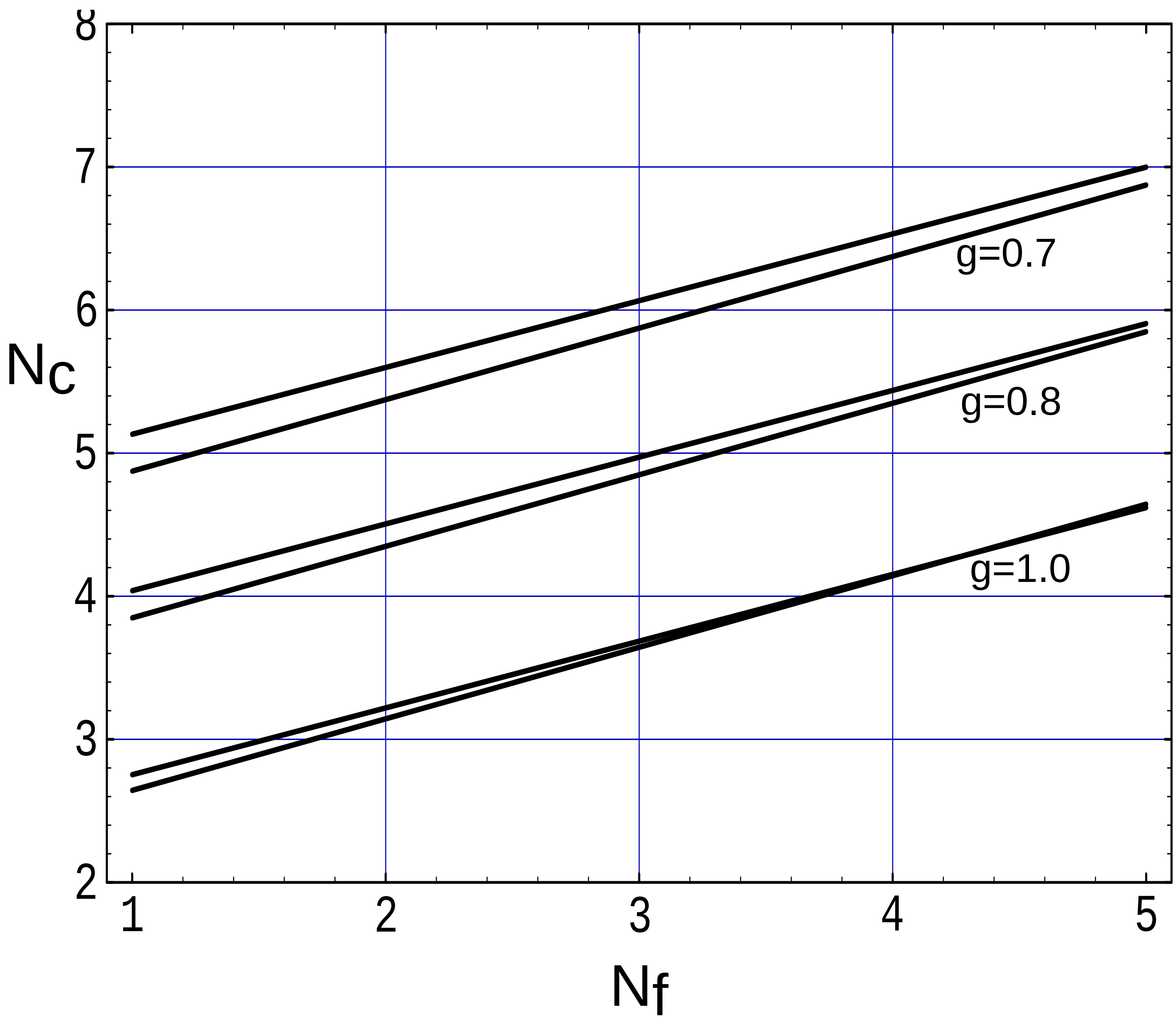

Figure 1: Constraints on hidden sector gauge group $SU(N_c)$ with N_f light flavors, such that the supersymmetry breaking scale ($m_{3/2}$) is between 100 GeV (bottom plots) and 1 TeV (top plots), and the hidden matter condensate scale is $\langle T\bar{T}\rangle/M = 10^{10}$ GeV, for different values of the gauge coupling (g) at the Planck scale.